https://doi.org/10.36007/5093.2024.321

ESTIMATING GEOMAGNETIC VARIATIONS PRODUCED BY A FIELD-ALIGNED CURRENT AT MIDDLE LATITUDES

Fridrich VALACH[1] – Eduard KOČI[2]

ABSTRACT
When interpreting geomagnetic variations during magnetic storms, Fukushima's theorem must sometimes be considered. According to the theorem, any magnetic effect from the longitudinal currents (f. a. c.) cannot be observed at the Earth's surface because that magnetic effect is cancelled by the Pedersen currents diverging or converging in the ionosphere. The assumptions of this theorem are: (1) the f. a. c. is perpendicular to the conducting ionospheric layer, and (2) the conductivity of the ionosphere is homogeneous. Typical places where the assumptions of this theorem are satisfied are the places on the planet at high geomagnetic latitudes (approximately said at high geographic latitudes). In the presented work, we study some consequences of Fukushima's theorem in middle latitudes. Our conclusions are limited to estimates of indicative character since we used certain simplifying assumptions.

KEYWORDS
geomagnetic field, field-aligned currents, Biot–Savart law, Fukushima's theorem, ionosphere

INTRODUCTION

In interpreting some rapid changes in the geomagnetic field, such as certain variations during magnetic storms, researchers (e.g., Tanaka et al., 2020) often refer to Fukushima's theorem (Fukushima, 1976). The theorem states that the magnetic field generated by an individual field-aligned current (f. a. c.) flowing from above into the ionosphere cannot be detected at the Earth's surface because the effect of such an f. a. c. is perfectly cancelled by Pedersen currents diverging in all directions in the plane of the conducting ionosphere. The assumed current system is shown in Figure 1. The theorem, however, requires two essential conditions to be satisfied: (1) the longitudinal current direction has to be perpendicular to the Earth's surface (i.e., to the conducting layer in the ionosphere), and (2) the conductivity of the ionosphere has to be uniform (and sufficiently high). The same effect, i.e., the cancellation of the longitudinal current effect by the Pedersen currents, applies to the upward longitudinal current; in this case, the Pedersen currents converge from all sides in the ionospheric layer and continue upward in the form of the f. a. c.

The necessary conditions of Fukushima's theorem can be almost perfectly satisfied on the dayside of the planet (due to sufficiently high atmospheric conductivity caused by solar radiation) at high geomagnetic latitudes (due to the nearly perpendicular direction at which the

[1]Mgr. Fridrich VALACH, PhD.; Geomagnetic Observatory of the Earth Science Institute, Slovak Academy of Sciences, Komárňanská 108, 947 01 Hurbanovo, Slovak Republic; email: geoffval@savba.sk
[2]Mgr. Eduard KOČI; Slovak Central Observatory, Komárňanská 137, Hurbanovo, Slovak Republic, and Geomagnetic Observatory of the Earth Science Institute, Slovak Academy of Sciences, Komárňanská 108, 947 01 Hurbanovo, Slovak Republic; email: eduard.koci@suh.sk

field lines of the Earth's magnetic field intersect the plane of the highly conductive layer of the ionosphere) (Tanaka et al., 2020). At mid-latitudes, such as the location of the Hurbanovo Observatory, the assumptions of Fukushima's theorem cannot be perfectly satisfied, even on the day side of the planet. The reason is that the magnetic inclination in this region of the Earth is about 65˚ (holds for Hurbanovo), which is quite different from the perpendicular direction.

Thus, it may be interesting, and perhaps even useful, to consider the impact of an individual f. a. c. entering (or leaving) the ionosphere at mid-latitudes. Such considerations, first proposed by Fukushima (1976), will be addressed in the present paper.

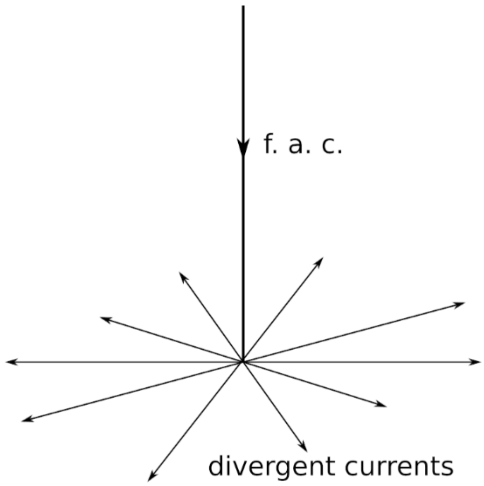

Figure 1 Sketch of a perpendicularly incident f. a. c. and divergent currents in the ionosphere

RESEARCH METHODOLOGY

In this paper, we investigated the implications of the imperfectly fulfilled conditions of the Fukushima’s theorem. The chosen procedure was as follows. Providing we limit our considerations only to obtain a reasonable estimate of the influence of an f. a. c. on the geomagnetic field at the Earth's surface, the task is not computationally demanding. We will divide the problem into two subtasks: in the first subtask, we will be interested in the effect of an f. a. c. of reasonable magnitude on the observations of the geomagnetic field at mid-latitude ground-based stations; we will perform the calculations for a particular value of the inclination of 65˚. In the second subtask, we will focus on the case where the f. a. c. enters the ionosphere near the boundary separating high and low conductivity.

The main part of our study was to express the magnetic field from an oblique f. a. c. partially cancelled by diverging currents. To formulate this problem mathematically, we need to notice that if the diverging Pedersen currents can entirely eliminate the magnetic effect of an f. a. c. in ground-based observations, this is a consequence of certain remarkable equality: the downward-directed f. a. c. generates magnetic fields at the Earth's surface exactly as large, but inversely oriented, as those generated by the set of diverging currents. It is natural to assume that the set of the diverging currents depends only on the magnitude of the f. a. c. and does not depend on the angle at which the f. a. c. is incident.

If the above ideas are valid, then the system consisting of the obliquely incident f. a. c. and diverging Pedersen currents may be replaced by a system consisting of that oblique f. a. c. and a perpendicularly rising f. a. c. (see Fig. 2). To quantify the geomagnetic variation, the following form of Biot-Savart law can be used, which describes the magnetic field **B**(**r**) at location **r** caused by the electric current *I* in a conductor that has the shape of a curve C (we consider an idealized conductor with negligible cross-section, the integration is along the curve C):

$$\mathbf{B}(\mathbf{r}) = (\mu_0 I/4\pi) \cdot \int_c R^{-3} \cdot d\boldsymbol{\ell} \times \mathbf{R} \,. \quad (1)$$

In this equation, $\mathbf{R} = \mathbf{r} - \mathbf{r}_\ell$, where $\mathbf{r}_\ell$ is the position vector of the infinitesimal length vector $d\boldsymbol{\ell}$ along the curve C, and μ_0 is the vacuum permeability.

The problem is fully solvable by means of mathematical analysis. To obtain the analytical results we used the computer algebra system Maxima, the stable release 5.47.0, available on the webpage (Maxima, 2024). Specifically, we used the functions *integrate* and *partfrac*; the former to obtain the indefinite integral of expression (1) and the latter to rewrite the complex resulting expression into a more convenient form, which we could work with analytically further.

Figure 3 outlines the geometry of the system of the currents required to solve the task. Let us consider only a limited region near the place on the Earth's surface that is located below the point the f. a. c. connects with the conducting layer of the ionosphere. This restriction enables us to approximate both the downwards f. a. c. and upwards f. a. c. by straight lines, as shown in the figure. If we were interested in a larger region on the Earth's surface, the approximation by the straight lines would not be sufficient. When integrating along the lines indicated in the figure (the two lines together form the curve C), one of the integration boundaries is the height of the conducting layer of the ionosphere above the Earth's surface, h_0 = 100 km (the conducting layer is approximated with a thin plane). The other integration boundary, denoted by h, is at a sufficient height above the conductive layer. The largest part of the f. a. c. effect on the magnetic field generated near the Earth's surface comes from the part of the electric current closest to the Earth's surface. We have verified that when we set the integrating boundary h to, for example, $h = 15h_0$, instead of infinity, the numerical result differs only in the third significant digit.

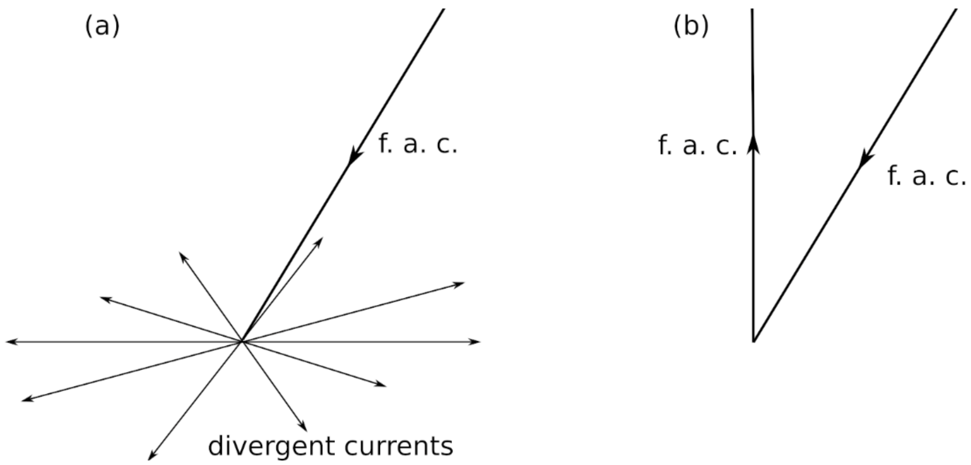

Figure 2 Sketch of two current systems that produce an identical magnetic field near the Earth's surface: (a) an obliquely downward flowing f. a. c. and divergent currents flowing in the ionosphere, (b) an obliquely downward and a vertically upwards flowing f. a. c.`s

In the calculations of Equation (1), besides the above geometry, it is necessary to enter a reasonable value of the electric current I. We will use the information from the work (Wang et al., 2006) for an approximate estimate of a reasonable value in the case of some extreme geomagnetic event. Wang and his team present the results of f. a. c. measurements by the

CHAMP satellite during geomagnetically disturbed conditions in October and November 2003. Estimating from an area of approximately 100 km × 100 km and the maximum measured electric current density, an indicative extreme current value is ~10^6 A; we will use this value in our modelling. It is good to remind the reader that the dependence of the variation of the magnetic field is directly proportional to the electric current (see Equation 1), therefore it is straightforward to transform our results to any other value of the current *I*.

We used the scientific programming language GNU Octave, version 9.2.0 (GNU Octave, 2024), to graphically display the results (Figs. 4 to 7) in the following section.

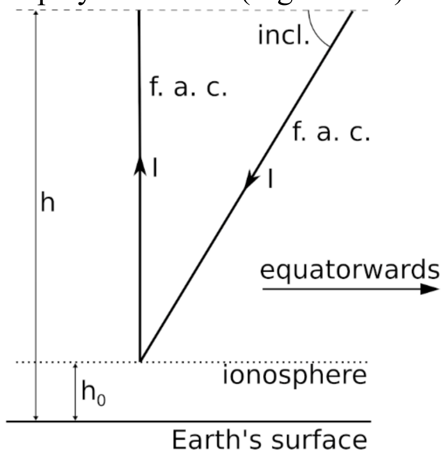

Figure 3 Data on the geometry of the current system for calculations with Equation (1)

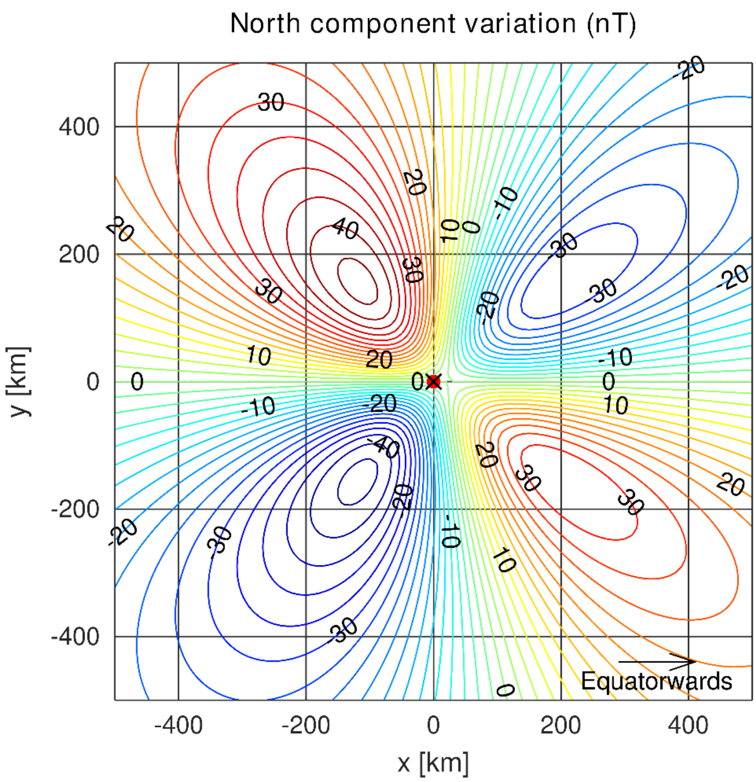

Figure 4 Contour map of the variation of the North component due to the system of the currents sketched in Figure 2b. A current of 10^6 A and an inclination of 65° were chosen in the model. The point highlighted in the centre of the image is located directly below the point where f. a. c. feeds the conductive layer in the ionosphere.

RESULTS

Figures 4 and 5 show the variation in the North and East components of the geomanetic field, respectively, caused by an f. a. c. with the above-described characteristics. For the definition of geomagnetic field components, see e.g. (Valach & Váczyová, 2016). The variation in the North component reaches 50 nT at most (Fig. 4). Thus, it is not a variation that would be comparable in size to the variations of the North component of the geomagnetic field typically observed in more intense magnetic storms (e.g., Gonzalez et al., 2011). We note that in the close vicinity (approximately ±100 km) from the point that is located below the point of connection of the f. a. c. to the conductive ionospheric layer, our results cannot accurately describe the reality. It is because, unlike the idealized very thin linear conductor, the region in which the current flows has rather the form of a cylinder with a cross-section of ~10^4 km^2.

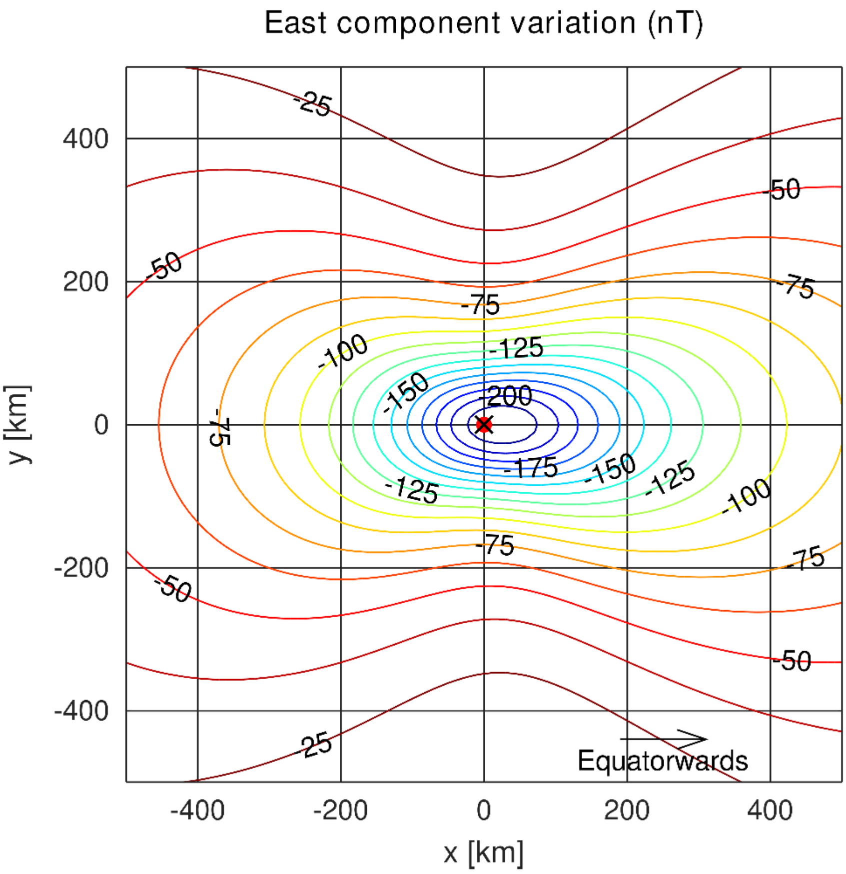

Figure 5 Contour map of the variation of the East component due to the system of the currents sketched in Figure 2b. A current of 10^6 A and an inclination of 65° were chosen in the model. The point highlighted in the centre of the image is located directly below the point where f. a. c. feeds the conductive layer in the ionosphere.

The variations of the East component of the field (Fig. 5) are comparable to the variations during intense storms. It concerns mainly the area equatorwards from the point beneath the place where the f. a. c. meets the ionosphere at altitude h_0. In this area, the East component generated by the f. a. c. pair goes under –150 nT. In the opposite direction, i.e. towards the pole, there is another interesting area in which the generated East component is less than –100 nT. However, the very low values displayed close to the centre of the figure cannot be interpreted as real. Bearing in mind the finite cross-section of the current, we must exclude from consideration the results obtained for the area in the ca. 100 km vicinity around the point mentioned above.

An interesting aspect for practical interpretations of magnetic variations is also the magnetic effect of an f. a. c. on the East component for a non-uniform ionosphere. The contours in Figures 6 and 7 show the variations of the magnetic field that would be observed at the Earth's surface if the effect of an oblique downward oriented f. a. c. with the parameters described in the previous section was not cancelled by the diverging currents in the ionosphere. Of course, such a case in its pure form cannot take place because the inflowing current must flow away somewhere. Nevertheless, this non-physical example will serve well for the following qualitative considerations. The variation of the North component would be significant: as expected, it would be negative east of the f. a. c. and would be positive west of the f. a. c., and its magnitude would exceed 250 nT, which is comparable to the variations that are typical in intense magnetic storms.

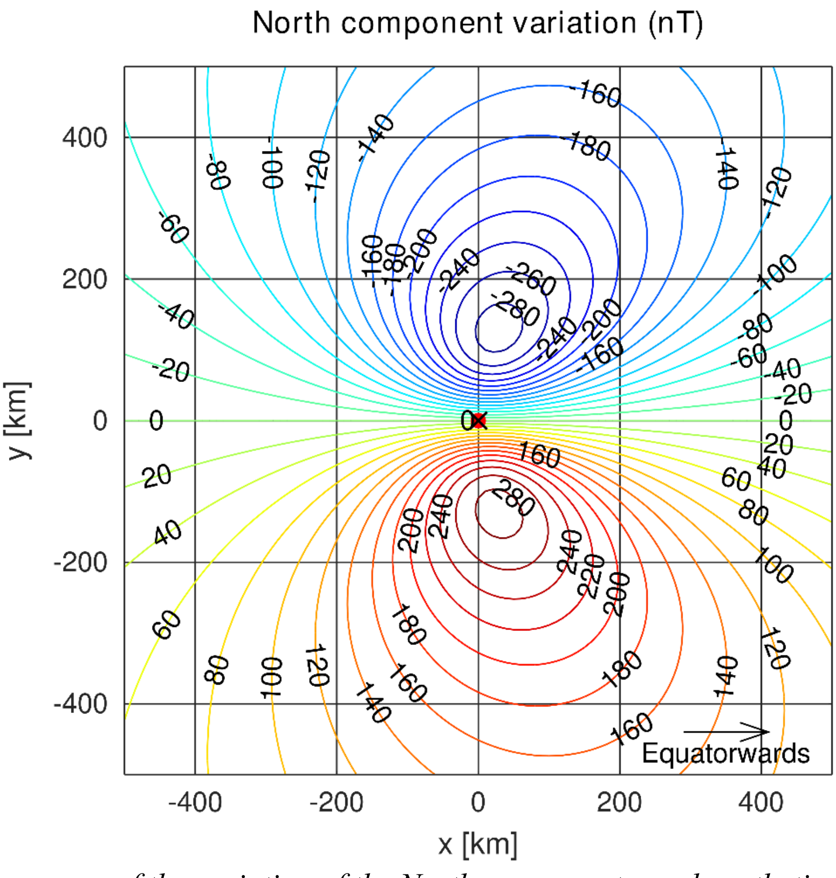

Figure 6 Contour map of the variation of the North component as a hypothetical consequence of the oblique downwards flowing f. a. c. if its magnetic field were not compensated by the divergent currents. A current of 10^6 A and an inclination of 65° were chosen in the model. The point highlighted in the centre of the image is located directly below the point where f. a. c. feeds the conductive layer in the ionosphere.

The East component would even decrease to less than –400 nT equatorward and exceed 100 nT poleward at a distance of 200 km from the location beneath the connection of the f. a. c. to the ionosphere.

Let us consider the case that part of the ionospheric layer (the one closer to the pole) would be highly conductive and the other part (closer to the equator) would be non-conductive. For example, the conductivity in the conductive part would be caused by the intense precipitation of particles in the aurora, limited only to this region. The boundary between these different areas would be formed by a straight line running from east to west. Let the f. a. c. flow into the conductive region at a point close to this boundary (Fig. 8). The in-flowing current could have to disperse only throughout the conducting region. If the divergent pattern were to be preserved, the currents would have to be twice as dense as in the case of uniform conductivity of the ionosphere. These diverging currents would not only compensate the magnetic field from the f. a. c. beneath the conductive ionosphere, but even outweigh it. The magnetic field from the currents in the conducting region would manifest itself below the conducting region, and even to a lesser degree, would manifest itself beneath the non-conducting region. The consequence would have to be an amplified negative variation of the East component of the geomagnetic field, both under the conductive and non-conductive part of the ionosphere.

Using the same reasoning, we can conclude that for an upward f. a. c., the variation of the East component would be positive. Eventually, similar considerations can be made for the exchanged conductive and non-conductive regions.

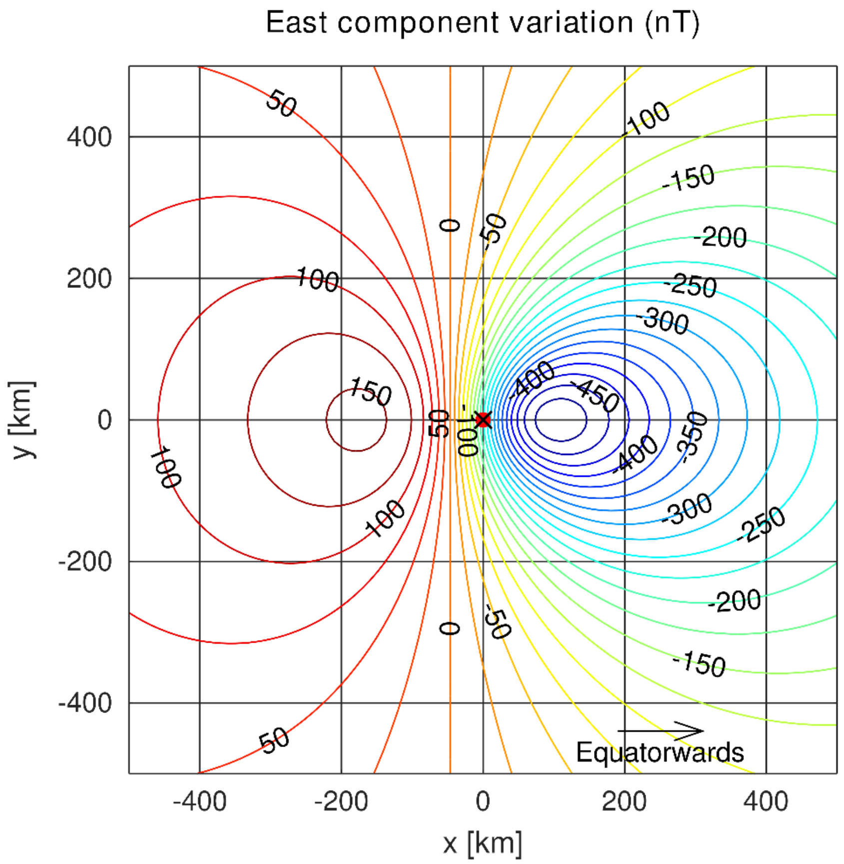

Figure 7 Contour map of the variation of the East component as a hypothetical consequence of the oblique downwards flowing f. a. c. if its magnetic field were not compensated by the divergent currents. A current of 10^6 A and an inclination of 65° were chosen in the model. The point highlighted in the centre of the image is located directly below the point where f. a. c. feeds the conductive layer in the ionosphere.

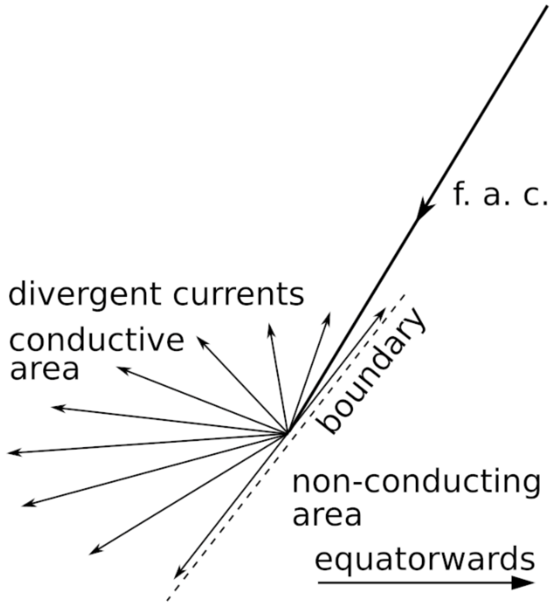

Figure 8 A sketch for qualitative considerations in the case of inhomogeneous conductivity of the ionosphere

CONCLUSIONS

We studied cases of magnetic variations generated by a simplified system of an oblique downwards oriented f. a. c. and diverging currents (here replaced by a vertical upwards flowing f. a. c.) in a homogeneous ionosphere. We also dealt with a similar current system when the ionosphere is not uniform. We considered the conditions valid in middle latitudes – we set the inclination equal to 65°. From the presented simple model, it follows that in the case of a uniformly conductive ionosphere, even for a very strong f. a. c., the variations of the North component are not large enough compared to typical extreme magnetic storms. On the other hand, the East component generated by the f. a. c. system could be about –150 nT (here, the minus sign means the variation in the west direction) for the f. a. c. of ~10^6A.

Also, we implied that the inhomogeneous conductivity of the ionosphere leads to large variations in the East component of the geomagnetic field; these might be variations comparable to those that occur in intense magnetic storms. Provided the input conditions we set in our simple model are not too far from the real conditions in the near-Earth environment, our conclusions can contribute to the interpretation of geomagnetic disturbances in mid-latitudes.

COMPLETION

FV devised the idea of the study, participated in the calculations and interpretations, drew the figures, and wrote the manuscript. EK participated in the calculations and interpretations and verified the results of all the calculations in the study.

ACKNOWLEDGEMENT

The research in this paper was supported by the VEGA Grant 2/0003/24 of the Scientific Grant Agency of the Ministry of Education of Slovak Republic and the Slovak Academy of Sciences. The authors thank the editor and reviewers for their careful reading of the manuscript and helpful comments.

REFERENCES

Fukushima, N. (1976). Generalized theorem for no ground magnetic effect of vertical currents connected with pedersen currents in the uniform conductivity ionosphere. *Report of Ionosphere and Space Research in Japan, 30*, pp. 35–40.

GNU Octave. (03.10.2024) *Scientific Programming Language*. https://octave.org/

Gonzales, W. D., Echer, E., Clúa de Gonzales, A. L., Tsurutani, B. T., & Lakhina, G. S. (2011) Extreme geomagnetic storms, recent Gleissberg cycles and space era-superintense storms. *J. Atmos. Sol.-Terr. Phys., 73*, pp. 1447–1453.
https://doi.org/10.1016/j.jastp.2010.07.023

Maxima. (03.10.2024) *Computer Algebra System*. https://maxima.sourceforge.io/

Tanaka, T., Ebihara, Y., Watanabe, M., Den, M., Fujita, S., Kikuchi, T., Hashimoto, K. K., & Kataoka, R. (2020). Reproduction of ground magnetic variations during the SC and the substorm from the global simulation and Biot-Savart's law. *Journal of Geophysical Research: Space Physics*, *125(2)*, Article e2019JA027172. https://doi.org/10.1029/2019JA027172

Valach, F. & Váczyová, M. (2016). A smartphone as a compass: accuracy of fixing direction according to the magnetic field. *Proceedings of the Conference of J. Selye University 2016*, Komárno, pp. 98–106. ISBN 978-80-8122-190-3.

Wang, H., Lühr, H., Ma, S. Y., Weygand, J., Skoug, R. M., & Yin, F. (2006). Field-aligned currents observed by CHAMP during the intense 2003 geomagnetic storm events. *Ann. Geophys., 24*, 311–324. https://doi.org/10.5194/angeo-24-311-2006